TABLE 1. CORRELATION CONSTANTS

	2-methyl propane	2-methyl butane	2-methyl pentane	2-methyl hexane*	2-methyl heptane*
A_o	10.233	13.470	14.930	15.799	16.603
B_o	0.1375	0.1584	0.1729	0.1856	0.1977
C_o	0.8499×10^6	1.4833×10^6	2.8500×10^6	4.9530×10^6	7.6430×10^6
a	1.9376	5.1429	7.4286	9.2778	11.001
b	0.04243	0.08333	0.1215	0.1643	0.2131
c	0.2860×10^6	0.8333×10^6	1.400×10^6	2.0314×10^6	2.7600×10^6
α	0.001074	0.001700	0.00235	0.00314	0.00413
γ	0.0340	0.0480	0.0620	0.0776	0.0960

* Extrapolated with correlation.

to 300°C.) for 2-methyl butane, and 0.16% (1.5 to 5.5 g. mole/liter; 250° to 275°C.) for 2-methyl pentane. Additionally calculated critical pressures differed by 0.220, 1.68, and 0.300 atm. from observed values for these compounds.

Extrapolated constants for 2-methyl hexane and 2-methyl heptane were used to calculate critical pressures for these compounds. These calculated pressures differed by 0.377 and 0.295 atm. respectively from the literature values for 2-methyl hexane (7) and 2-methyl heptane (8).

Only the constants for 2-methyl propane (isobutane) were fitted to the two-phase region. The remaining constants were fitted only to the superheated vapor region. It is not recommended that they be used in the estimation of properties for the vapor-liquid dome.

It is difficult to evaluate correlations of this type because they afford methods of estimating compressibility data which cannot be measured easily or at all. However it is apparent that the correlation does reproduce the compressibility data and critical points of the base compounds with good precision. Furthermore good agreement is obtained with extrapolated correlation constants for the critical points of 2-methyl hexane and 2-methyl heptane.

On the basis of this analysis it is recommended that the correlation be used to estimate higher molecular weight 2-methyl paraffins compressibility data. The procedure recommended in the paper of Canjar, Smith, Volianitis, Galluzzo, and Cabarcos (*1*) should be used for compounds higher in molecular weight than 2-methyl heptane (no literature critical temperatures are given for such compounds). This procedure is as follows:

1. Estimate roughly the critical temperature of the compound.
2. Calculate the Benedict-Webb-Rubin-Friend equation constants from the correlation equations.
3. Calculate pressure as a function of density at the assumed critical point.
4. If the slope is zero at the isotherm's inflection point, then the assumed critical temperature is correct. If it is not, another temperature should be assumed, and steps 1 to 4 should be repeated.

NOTATION

A_o, B_o, C_o, a, b, c, α, γ = constants of the Benedict-Webb-Rubin-Friend equation of state
Tc = critical temperature
Tc' = critical temperature of 2-methyl propane
$C'o$ = co-constant for 2-methyl propane
N_s = number of carbon atoms in the straight chain
N_t = total number of carbon atoms

LITERATURE CITED

1. Canjar, L. N., R. F. Smith, E. Volianitis, J. F. Galluzzo, and M. Cabarcos, *Ind. Eng. Chem.*, **47**, 1028 (1955).
2. Benedict, Manson, G. B. Webb, and L. C. Rubin, *Chem. Eng. Progr.*, **47**, 419 (1951).
3. Kelso, E. A., and W. A. Felsing, *J. Am. Chem. Soc.*, **62**, 3132 (1940).
4. Griskey, R. G., and L. N. Canjar, *A.I.Ch.E. Journal*, **5**, 29 (1959).
5. Isaacs, R., K. Li, and L. N. Canjar, *Ind. Eng. Chem.*, **46**, 199 (1954).
6. Young, S., *Proc. Phys. Soc. (London)*, 13, 602 (1894).
7. Kay, W. B., and J. McMicking, *Am. Petrol. Inst. Research Rept.*
8. "American Petroleum Institute Research Project No. 44," Carnegie Inst. Technol., Pittsburgh, Pennsylvania.

Manuscript received September 11, 1961; revision received July 24, 1962; paper accepted July 30, 1962. Paper presented at A.I.Ch.E. Denver meeting.

Effect of Natural Convection Instabilities on Rates of Heat Transfer at Low Reynolds Numbers

GEORGE F. SCHEELE and THOMAS J. HANRATTY

University of Illinois, Urbana, Illinois

Variation of the density and viscosity of a fluid in a pipe due to temperature variation can affect the heat transfer coefficient h because of changes in the velocity profile and because of transition to an unsteady flow if the distortion of the velocity profile is sufficiently large. For water these effects will primarily be due to density variations. This paper presents experimental data on the effect of unsteady flow on the heat transfer coefficient for flow of water in a vertical pipe. Two types of heat transfer experiments are conveniently conducted, one with a constant heat flux at the wall and the other with a constant temperature wall. When heating or cooling with a constant temperature wall the difference between the wall temperature and the bulk fluid temperature ΔT is changing throughout the heat transfer section, and therefore the effect of the heat transfer upon the flow field is changing as well. When heating with a constant flux, if the changes in temperature are affecting only the density appearing in the gravity term

George F. Scheele is at Cornell University, Ithaca, New York.

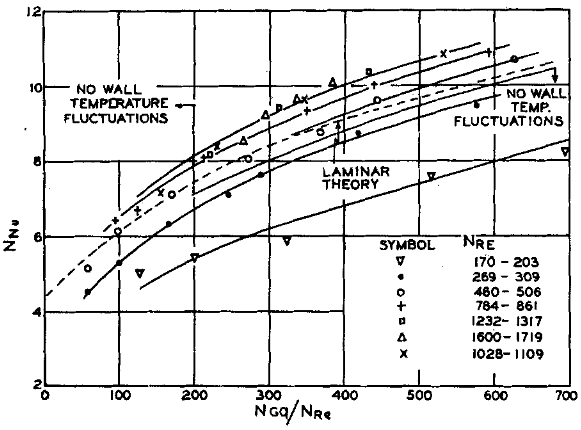

Fig. 1. Disturbed flow heat transfer coefficient for upflow constant flux heating.

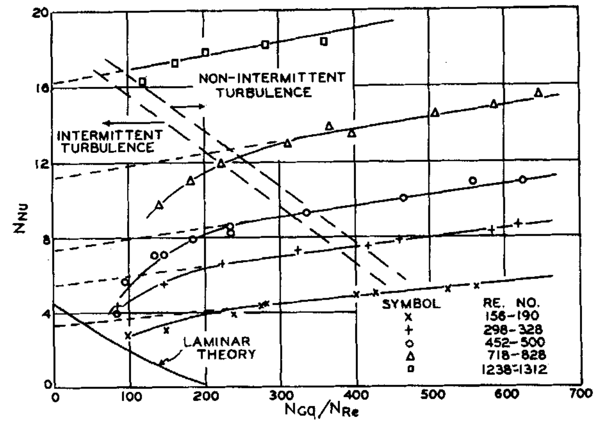

Fig. 2. Disturbed flow heat transfer coefficient for downflow constant flux heating.

of the equations of motion, a condition is attained far downstream in the heat transfer section such that there is no further change in the velocity field and such that the time average temperature and pressure are varying linearly with downstream distance. The experiments reported in this paper used a long enough section of electrically heated pipe that a fully developed flow existed downstream. Therefore if the flow became unsteady in the heat transfer section, the effect of this unsteady flow on the rate of heat transfer could be examined over a length of pipe in which ΔT is constant. By performing both upflow and downflow experiments the cases of gravity with and opposite the forced flow were examined.

An analytical solution for the fully developed temperature and velocity fields for symmetrical laminar flow has been presented by Hanratty, Rosen, and Kabel (*3*), by Hallman (*1*), and by Morton (*4*). The shape of the velocity and temperature fields is a function of the ratio of the heat flux to the flow rate as represented by the ratio of two dimensionless groups N_{Gq}/N_{Re}. If the field is steady and laminar, an increase in N_{Gq}/N_{Re} for upflow causes an increase in the velocity gradient at the wall and therefore an increase in the heat transfer coefficient, while for downflow an increase in N_{Gq}/N_{Re} causes a decrease in the velocity gradient at the wall and a decrease in the heat transfer coefficient.

Results of studies of the stability of these flows have been reported in a number of papers (*2, 3, 5, 6*). It has been found that the stability of the flow depends primarily on the shape of the velocity profile and only secondarily on the value of the Reynolds number, if at all. For upflow heating the flow first becomes unstable when the velocity profiles develop points of inflection ($N_{Gq}/N_{Re} > 32.94$), in agreement with Lord Rayleigh's theorem on the stability of laminar velocity profiles (*7*). Transition to an unstable flow involves the gradual growth of small disturbances, and therefore it is quite possible to have unstable flows without observing transition. For downflow heating the flow instability is associated with separation at the wall. Transition to an unsteady flow is sudden and therefore occurs shortly after an unstable flow occurs. Over the range of N_{Re} of 80 to 4,800 transition occurs for values of N_{Gq}/N_{Re} of 78 to 56 for downflow.

DESCRIPTION OF EXPERIMENTS

Experiments were conducted in a 60 ft. vertical length of 0.787 in. I.D. copper pipe. The downstream 50 ft. were wrapped with Chromel-A heating ribbon and surrounded with insulation. There were five independent 10 ft. heating sections. The maximum length of heat transfer section was 762 pipe diameters, and the length of the calming section was 152 to 762 pipe diameters, depending on the number of heat transfer sections used. The entry and the feed system were designed so that they introduced very little disturbance into the pipe, and it was possible to obtain isothermal laminar flow for N_{Re}* values in excess of 5,000. The heat input to the water was determined from an overall heat balance with measured inlet and outlet water temperatures. Temperature fluctuations in the fluid leaving the heat transfer section were detected by amplifying the signal from a 30 gauge copper-constantan thermocouple located at the pipe axis. Wall temperature measurements were made along the entire length of heated pipe with 30 gauge copper-constantan thermocouples imbedded in the wall of the pipe. In runs where there were fluctuations in the measured wall temperature, an average of ten readings taken at 15-sec. intervals were used. The magnitude of the fluctuations of the wall temperature were not as large as those obtained by Hallman (*2*), nor was any appreciable asymmetry in wall temperatures noted as was the case in Hallman's downflow experiments. The radial temperature differences were smaller in the present experiments than in the experiments of Hallman. Likewise the thick pipe wall of large thermal conductivity would tend to even out asymmetries if they were present and to dampen temperature fluctuations. Since a constant heat flux was applied to the system and heat losses were small, the bulk fluid temperature increased linearly throughout the heat transfer section. If fully developed velocity and temperature fields existed over a length of the heat transfer section close to the outlet, the wall temperature in this region also varied linearly with the same rate of increase as the bulk temperature, the difference between the bulk and the wall temperature being constant. For runs in which a fully developed unsteady flow existed, a Nusselt number N_{Nu} was calculated from the difference between the wall temperature and the bulk temperature and a heat balance on the system. Fluid properties used for calculating N_{Nu}, N_{Gq}, and N_{Re} were evaluated at the calculated average temperature which existed at the midpoint of the fully developed flow region used for obtaining the average radial temperature difference.

Details of these experiments and tables of results are presented in a thesis by one of the authors (*5*).

* The Reynolds number used in this paper is based on the radius of the pipe.

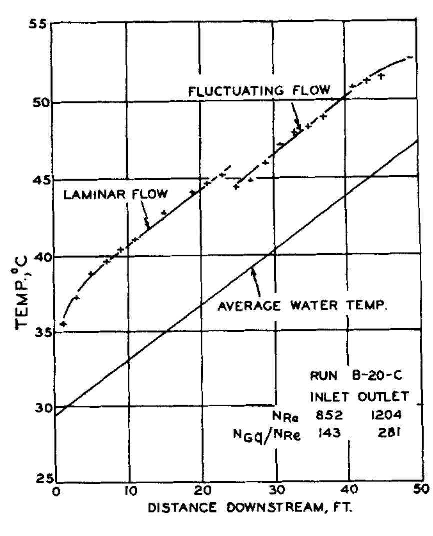

Fig. 3. Wall temperature behavior when transition occurs for upflow constant flux heating.

RESULTS

The character of the unsteady flow, as indicated by temperature fluctuation measurements in the fluid and in the pipe wall and by watching the motion of a dye plume (6), was quite different depending on whether the flow was up or down. For heating in upflow these measurements indicated an oscillating flow but did not show the random structure characteristic of turbulence unless the Reynolds number was close to the value of 1,050 needed to sustain a turbulent field for isothermal flow. However this observation might just reflect the insensitivity of the temperature sensing instrument. For heating in downflow the transition to unsteady flow at all Reynolds numbers was noted by patches of turbulent and nonturbulent flow. The frequency of appearance of these turbulent patches increased with increasing N_{Gq}/N_{Re} until the flow appeared completely random. Detailed descriptions of these unstable flows are presented in a thesis by one of the authors (6).

Measured Nusselt numbers for fully developed unsteady flows are shown in Figures 1 and 2. The marked differences between the upflow and downflow data reflect the differences in the character of the unsteady flow discussed above. For upflow heating N_{Nu} is larger than what would exist for steady parabolic flow. However except for the data at low Reynolds numbers the Nusselt numbers are not too different from those predicted for distorted laminar flow, which are already quite large. For laminar upflows at $N_{Gq}/N_{Re} > 32.94$, the velocity profiles have a dimple at the tube center, and the point of maximum local velocity moves radially toward the tube wall as the ratio of heat flux to flow rate is increased. The laminar theory predicts a reversal of flow in the center of the tube for $N_{Gq}/N_{Re} > 319.1$. This displacement of the maximum in the velocity profile toward the tube wall causes the increase in the velocity gradient at the wall which accounts for the large Nusselt numbers predicted by laminar theory. Since such highly distorted average velocity profiles might not exist for unsteady flows, it should not be surprising that unsteady flows at low Reynolds numbers can have lower Nusselt numbers than are predicted by laminar theory.

For downflow heating the measured values of Nusselt numbers are much greater than those predicted for distorted laminar flow. Increases in N_{Gq}/N_{Re} and N_{Re} cause N_{Nu} to increase; however the effect of N_{Re} appears to be much greater for downflow than for upflow heating. Since laminar flow heat transfer coefficients for downflow are smaller than for upflow, transition to disturbed flow has greater effect on the heat transfer coefficients for downflow. This is illustrated by the wall temperatures shown in Figures 3 and 4, where transition to an unsteady flow is evident by a much larger drop in the wall temperature for downflow heating.

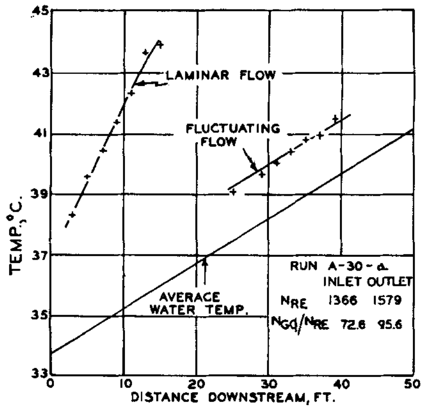

Fig. 4. Wall temperature behavior when transition occurs for downflow constant flux heating.

Regions of intermittant and nonintermittant turbulence are indicated in Figure 2 for downflow heating. In the region of nonintermittant turbulence the variation of N_{Nu} with N_{Gq}/N_{Re} can be represented by a linear relation. This linear relation has been extrapolated to $N_{Gq}/N_{Re} = 0$ in order to compare the measurements with heat transfer relations obtained at high Reynolds numbers under the condition that natural convection was not affecting the heat transfer. A cross plot of Figure 2 is shown in Figure 5, where the line for $N_{Gq}/N_{Re} = 0$ represents the extrapolation of the measurements. It is seen that the line for $N_{Gq}/N_{Re} = 0$ can be represented by an equation similar to Dittius-Boelter equation for turbulent flow heat transfer at high Reynolds numbers.

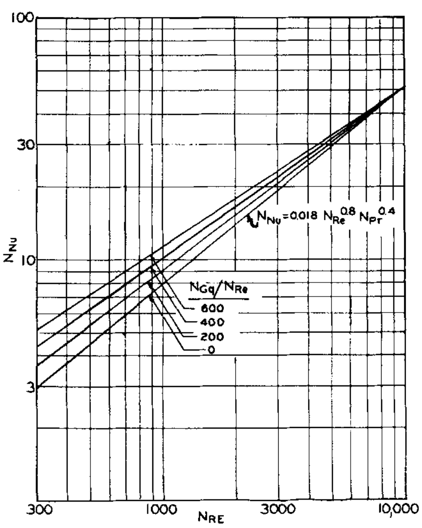

Fig. 5. Heat transfer coefficients for constant flux downflow heating.

If the measurements shown in Figures 1 and 2 are to be compared with heat transfer data with a constant temperature wall, it is convenient to use a Grashof number N_{Gr} defined by the relation $N_{Gr} = N_{Gq}/N_{Nu}$. Since the ΔT in N_{Nu} will be varying throughout the heat transfer section, so will N_{Gr}. However one must be cautious in applying these results to the case of a constant temperature wall. Since, for natural convection in the direction of flow, a considerable length of heat transfer section is needed for an unstable flow to develop a measurable unsteadiness, it is quite possible to have steady laminar flows at local conditions comparable to those covered by Figure 1.

ACKNOWLEDGMENT

Financial support for this work in the form of fellowship grants from Consolidation Coal Company and the National Science Foundation and a research grant from E. I. duPont de Nemours and Company, Incorporated, is acknowledged.

NOTATION

a = pipe radius
g = acceleration of gravity
h = heat transfer coefficient = $q/\Delta T$
k = thermal conductivity
N_{Gq} = $a^4 \beta g q / k \nu^2$
N_{Gr} = Grashof number = $a^3 \beta g \Delta T / \nu^2$
N_{Nu} = Nusselt number = ha/k
N_{Re} = Reynolds number = aV/ν
q = rate of heat transfer per unit area
T_W = wall temperature
T_{AV} = mixed average fluid temperature
ΔT = $T_W - T_{AV}$
V = average fluid velocity
β = coefficient of expansion of the fluid
ν = kinematic viscosity of the fluid

LITERATURE CITED

1. Hallman, T. M., *Trans. Am. Soc. Mech. Engrs.*, **78**, 1831 (1956).
2. ———, *No. A.S.A. Tech. Note D-1104* (1961).
3. Hanratty, T. J., E. M. Rosen, and R. L. Kabel, *Ind. Eng. Chem.*, **50**, 815 (1958).
4. Morton, B. R., *J. Fluid Mech.*, **8**, 227 (1960).
5. Scheele, G. F., Ph.D. thesis, Univ. Ill., Urbana, Illinois (1962).
6. ———, E. M. Rosen, and T. J. Hanratty, *Can. J. Chem. Eng.*, **38**, 67 (1960).
7. Schlichting, Hermann, "Boundary Layer Theory," 4 ed., p. 389, McGraw-Hill, New York (1960).

Manuscript received April 11, 1962; revision received August 6, 1962; paper accepted August 9, 1962.